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Deconstructing $(2,0)$ and Little String Theories

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Abstract: We argue that two four-dimensional strongly coupled superconformal field theories, on the Higgs branch in certain large N limits, become respectively $(2,0)$ theory and $(1,1)$ little string theory in six dimensions. We identify the spectrum of states responsible for the generation of the two extra dimensions and string winding modes. We establish the equivalence using orbifold realizations of the field theories and exploiting string dualities. We also speculate on deconstructions of M-theory.

Keywords: Field Theories in Higher Dimensions, Lattice Quantum Field Theory, String Duality.

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1. Introduction

Gauge theories in greater than four dimensions are of interest to phenomenologists and string theorists. Recently a new way of realizing higher dimensional theories was introduced [1] (for related work see [2]). These models are four-dimensional gauge theories defined on a "theory space"—"moose" [3] or "quiver" [4] diagrams constructed of sites and links—representing the gauge theory degrees of freedom. Although the dynamics of these models is four-dimensional at high energies, they appear higher-dimensional below some energy scale. This energy scale is set by the expectation values of the link fields, which higgs the gauge groups to the diagonal subgroup, allowing the fields to "hop" in the extra dimensions. In this low energy regime the theory space becomes a picture of the extra dimensions, and the effective action becomes that of a higher-dimensional gauge theory with latticized extra dimensions. In reference [1] both non-supersymmetric and $\mathcal{N} = 1$ supersymmetric models which develop one circular extra dimension were described. The massive gauge multiplets from the Higgs mechanism in the four-dimensional theory are the massive Kaluza-Klein (KK) modes of the compactified five-dimensional gauge theory.

The supersymmetric case is particularly interesting [1, 5]. The theory space for this example is a circle with N SU(k) gauge groups, all with gauge coupling G, connected by chiral superfields transforming as bi-fundamentals under the gauge groups at adjacent sites. The extra dimension is formed along the Higgs branch of the theory, where all chiral multiplets have the same expectation value $\Phi\mathbf{1}$. The parameters of the resulting effective latticized five-dimensional gauge theory are perturbatively related to those of the four-dimensional theory as:

$$g_5^2 = \frac{G}{\Phi}\,, \qquad 2\pi R = \frac{N}{G\Phi}\,, \qquad a = \frac{1}{G\Phi} = \frac{2\pi R}{N} \tag{1.1}$$

where a is the effective lattice spacing in the 5th dimension. At distances large compared to a the resulting five-dimensional gauge theory is both approximately five-dimensionally Lorentz invariant and $\mathcal{N} = 1$ supersymmetric, with corrections proportional to positive powers of the lattice spacing a. Both of these symmetries are *larger* than the original Lorentz invariance and supersymmetry in the four-dimensional model; in particular the supersymmetry has effectively doubled from 4 to 8 supercharges at low energies.

This theory provides a UV completion of a five-dimensional gauge theory by reverting to four-dimensional behavior at distances shorter than a. It is tempting to try to take $a \to 0$, while holding g_5^2 and R fixed—in this limit the theory would be an interacting five-dimensional, Lorentz invariant $\mathcal{N} = 1$ supersymmetric theory at *all* distances, providing a purely four-dimensional definition of a truly five-dimensional theory. However note that from (1.1) this limit implies that G, the four-dimensional Yang-Mills coupling, must be taken to infinity. But once G gets of order 4π the four-dimensional gauge theories are better described in the confining phase rather than the Higgs phase. The resulting dynamics no longer describes a five-dimensional theory. This limits the effective lattice spacing $a > g_5^2$ and prevents completion to a purely higher-dimensional theory.

This analysis suggests a higher-dimensional completion might be possible in a four-dimensional theory which remains in the Higgs phase even at strong coupling. If instead of the asymptotically free four-dimensional gauge theories of the previous example we had started with superconformal theories, we might hope to take the limit $G \to \infty$ without encountering a phase transition. This is the starting point for our present analysis: the identification of superconformal theory spaces which, in the Higgs phase, deconstruct extra dimensions.

We will find that the $\mathcal{N} = 2$ version of the previous example is such a superconformal theory. Although the massive gauge boson spectrum suggest a five-dimensional gauge theory, surprisingly we will find a non-perturbative set of monopoles and dyons which fill out the spectrum for a *six* dimensional theory. Similarly we will consider a superconformal theory space which is the triangulation of a torus. The massive gauge boson spectrum reproduces the KK modes of a six dimensional theory directly; remarkably the non-perturbative spectrum exhibits winding modes!

In the continuum limit described above these theories become well-defined interacting six dimensional Lorentz invariant theories with 16 supercharges. Only two such theories are thought to exist: the $(2,0)$ superconformal theory and little string theory. The $(2,0)_k$ superconformal field theory (SCFT), defined as a 5+1-dimensional conformal theory with a chiral $(2,0)$ supersymmetry in six dimensions [6], can be understood as a low energy description of k coincident M5-branes in M-theory or a decoupled description of the A_{k-1} singularity in type-IIB string theory. The $(2,0)$ superconformal field theory is useful in defining the matrix model [7] for M-theory on T^4. Seiberg realized that the matrix model for M-theory on T^5 and $T^5/\mathbb{Z}_2$ must be formulated in terms of a new six-dimensional "non-local" quantum theory called little string theory (LST) [8, 9]. LST with a $(1,1)$ SUSY can be found as a decoupled $g_s \to 0$ description of k coincident NS5-branes of type-IIB string theory with α' kept fixed while the T-dual $(2,0)$ LST is found on NS5-branes in type-IIA string theory. Matrix model descriptions of these theories have also been constructed [10, 11].

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While many insights have been gained about the nature of these six-dimensional theories, they remain mysterious and exhibit surprising properties. For instance, the holographic AdS/CFT dual [12] of the $(2,0)_k$ SCFT for k large describes M-theory in the $AdS_7 \times S^4$ geometry while the dual description of little string theories involves a linear dilaton background [13]. An interesting topological term in the low-energy derivative expansion of the equations of motion of the $(2,0)$ theory [14] underlies the description of the "tensionless strings" [15] that gain a nonzero tension on the Coulomb branch. While the $(2,0)$ SCFT exhibits phenomena connected with tensionless strings, the little strings have tension equal to the tension of "big strings", a Hagedorn density of states, and can be thought of as toy models for some general features of string theory while avoiding the complications of quantum gravity.

It is natural to suspect that our theories are a four-dimensional lagrangian description of $(2,0)$ and little string theory. We will argue this result by realizing our four-dimensional field theories as the $l_s \to 0$ limits of D3-branes on orbifolds. Using conventional S- and T-dualities we will show that our continuum limit precisely reproduces the definitions of $(2,0)$ SCFT and $(1,1)$ little string theory. Our theories at finite effective lattice spacing offer a discretization of $(2,0)$ and little string theory.

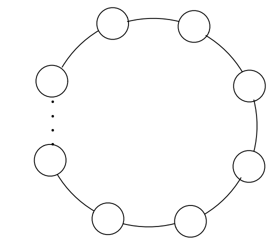

Figure 1: Theory space for an $\mathrm{SU}(k)^N$ model.

We will begin by describing our two superconformal theories that perturbatively deconstruct extra dimensions. To motivate our string construction we will describe the expected spectra of elementary and solitonic excitations. These states will have a standard interpretation in our orbifold constructions.

2. Superconformal deconstruction and spectra

Following the methods of [1] we construct a conformal four-dimensional $\mathcal{N}=2$ gauge theory whose theory space—that is the space of the moose/quiver diagram—is a circle with N sites, as shown in figure 1. Each site represents an $\mathrm{SU}(k)$ gauge group with gauge coupling G, while each link represents a hypermultiplet transforming as a bi-fundamental under the gauge groups at the link ends. It is a superconformal field theory, and is expected to have an $\mathrm{SL}(2,\mathbb{Z})$ duality [16]. This theory has a moduli space of vacua, including the direction where each hypermultiplet has an expectation value $\Phi\mathbf{1}$. As in [1] a fifth dimension is generated at low energies, where the action takes the form of a five-dimensional $\mathcal{N}=2$ gauge theory with a latticized fifth dimension. Perturbatively the effective lattice spacing is $a = 1/(G\Phi)$, the radius of the new dimension is $2\pi R_5 = Na = N/(G\Phi)$ and the five-dimensional gauge coupling is $g_5^2 = G/\Phi$. The spectrum of massive gauge bosons is

$$m_n^2 = 4G^2\Phi^2 \sin^2\frac{\pi n}{N} = G^2\Phi^2|e^{2\pi i n/N} - 1|^2 \tag{2.1}$$

precisely that of a latticized KK tower. These states are not BPS. There are no BPS states on the Higgs branch of $\mathcal{N}=2$ theories. So these masses could receive quantum corrections at strong coupling. However they are "almost BPS". That is they become BPS states in the low energy $\mathcal{N}=4$ theory. Since our states have masses that put them in $\mathcal{N}=4$ up to

$1/N$ deviations all quantum corrections to the masses must go to zero as $1/N$. Far from the top of the tower, $n \ll N$, the mass spectrum is nearly linear $m_n \simeq n/R_5$, reflecting the nearly continuum physics at distances large compared to the lattice spacing a. Since $\mathcal{N} = 2$ supersymmetry in 5 dimensions has 16 supercharges, while our four-dimensional field theory only has 8 supercharges, we have doubled the supersymmetry at distances large compared to the lattice spacing a. This is not surprising: the effective action for a latticized fifth dimension necessarily breaks these extra supersymmetries, but this breaking vanishes at distances where the five-dimensional continuum description applies. At the very lowest energies beneath the lightest massive KK mode, $E \ll 1/R_5$, the theory reduces to a four-dimensional $\mathcal{N} = 4$ SU(k) gauge theory with gauge coupling $g_4^2 = G^2/N = g_5^2/(2\pi R_5)$. As usual the compactification of a five-dimensional $\mathcal{N} = 2$ gauge theory on a circle leads to a four-dimensional $\mathcal{N} = 4$ gauge theory beneath the compactification scale.

In addition to the almost BPS spectrum of massive gauge bosons, this theory has a spectrum of "dyonic" excitations. We can infer the full spectrum by exploiting the SL$(2, \mathbb{Z})$ S-duality of the $\mathcal{N} = 2$ conformal field theory. The strong-weak duality is generated by $G \leftrightsquigarrow N/G$ (which interchanges $g_4^2 \leftrightsquigarrow 1/g_4^2$ in the very low energy $\mathcal{N} = 4$ theory). Therefore there are magnetic states with masses

$$M_n^2 = \frac{4N^2\Phi^2}{G^2} \sin^2 \frac{\pi n}{N} \qquad n \neq 0\,. \tag{2.2}$$

Remarkably this is the KK spectrum for yet another dimension of radius $2\pi R_6 = G/\Phi = g_5^2$. The S-duality can be rephrased directly as the transformation $R_5 \leftrightsquigarrow R_6$; in this form the existence of the 6th dimension is clear beyond the spectrum. The KK modes with momenta in both the fifth and sixth directions are dyons.

For distances large compared to a our quiver theory behaves like a 6 dimensional theory with 16 supercharges. Aside from the lattice spacing, the only breaking of conformal invariance is the compactification. Extrapolating these perturbative results to $G, \Phi \to \infty$ while keeping R_5, R_6 fixed shrinks the lattice spacing $a \to 0$, yielding an interacting conformal theory with the same BPS spectrum as $(2,0)_k$ SCFT compactified on a torus. In the next section we will show that this limit of our quiver theory *is* this $(2,0)_k$ SCFT. For finite $a \ll R_5, R_6$ our four-dimensional quiver field theory provides a discretization of $(2,0)$ theory. The fluctuation of each link vev is a modulus that corresponds to a fluctuation of the effective lattice spacing; we expect these moduli to decouple in our limit where the lattice spacing is taken to zero [17]. (These moduli are not apparent in the classical D-brane probe picture presented in the next section because there, the gauge group on the sites is U(k) rather than SU(k), and the additional moduli are lifted by the U(1) D-terms).

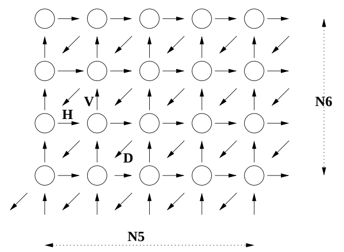

Figure 2: Theory space for the $\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ quiver gauge theory.

Another four-dimensional superconformal theory space is shown in figure 2. This is an $\mathcal{N} = 1$ gauge theory with $N_5 \times N_6$ SU(k) gauge groups at the sites [18, 19, 20]. The links are chiral superfields transforming as bi-fundamentals under the gauge groups at their

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ends. In addition we include a cubic superpotential for each triangle: $W = \pm\sqrt{2}\lambda\, tr\, HVD$ where the plus sign is for clockwise oriented triangles and the minus sign for anti-clockwise orientation. This theory has a line of superconformal fixed points in the (G, λ) plane [21]. Perturbatively this begins as the straight line $\lambda = G$. This theory has a large moduli space of vacua, including the direction where each H and V have expectation values $\Phi_5\mathbf{1}$ and $\Phi_6\mathbf{1}$ respectively, while D vanishes. In this case two extra dimensions are generated at low energies, where the action takes the form of a six-dimensional $\mathcal{N} = (1,1)$ gauge theory on a rectangular torus with two latticized dimensions. The effective lattice spacing is again $a_{5,6} = 1/(G\Phi_{5,6})$, the radii of the new dimensions are $2\pi R_5 = N_5 a_5 = N_5/(G\Phi_5), 2\pi R_6 = N_6 a_6 = N_6/(G\Phi_6)$ and the six-dimensional gauge coupling is $g_6^2 = 1/(\Phi_5\Phi_6)$. We can make a non-rectangular torus by choosing different directions in moduli space where all three of H, V, D acquire expectation values. The potential following from the superpotential includes "plaquette" operators which produce the $tr\, F_{56}^2$ pieces of the six-dimensional gauge action. The spectrum of massive gauge bosons is that of a latticized toroidal KK tower with mass scales $m_{5,6} = 1/R_{5,6}$. Since $\mathcal{N} = (1,1)$ supersymmetry in 6 dimensions has 16 supercharges, while our four-dimensional field theory only has 4 supercharges, we have quadrupled the supersymmetry at distances large compared to a. At the very lowest energies beneath the lightest KK mode, the theory reduces to a four-dimensional $\mathcal{N} = 4$ SU(k) gauge theory with gauge coupling $g_4^2 = G^2/(N_5N_6) = g_6^2/(4\pi^2 R_5 R_6)$.

In addition to the spectrum of massive gauge bosons, this theory has a spectrum of dyonic excitations. We can again infer the spectrum by exploiting S-duality. Making the transformation $G \leftrightsquigarrow N_5N_6/G$, along with the geometric symmetry $R_5 \leftrightsquigarrow R_6$, this takes $m_{5,6} = 1/R_{5,6} \to M_{5,6} = R_{5,6}/g_6^2$. This is a T-duality, and the dyonic spectrum includes "winding" modes!

For distances large compared to $a_{5,6}$ our quiver theory behaves like a 6 dimensional theory with 16 supercharges. In addition to the lattice spacing, and the compactification, this theory breaks conformal invariance through g_6^2. Taking $G \to \infty$ while keeping R_5, R_6 and g_6^2 fixed shrinks the lattice spacings $a_{5,6} \to 0$, yielding an interacting theory with winding modes compactified on a torus, just like the $(1,1)$ little string theory. In the next section we will show that this limit of our quiver theory *is* $(1,1)$ little string theory, with little string tension $1/g_6^2$. Again for finite $a_{5,6} \ll R_5, R_6, g_6$ our four-dimensional quiver field theory provides a discretization of $(1,1)$ little string theory.

3. String theory analysis

3.1 Orbifolds for quivers

Because the "latticized" gauge theories in $d > 4$ will be related to orbifolds in string theory, it is useful to review how the quiver gauge theories appear. Our presentation will be compressed: a complete account can be found in the original paper by Douglas and Moore [4]. The relevant orbifolds are $\mathbb{C}^3/\mathbb{Z}_N$ for the case of the $(2,0)$ superconformal field theory and $\mathbb{C}^3/\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ for the case of the $(1,1)$ little string theory. The former orbifold can be obtained from the latter by setting $N_5 = N$ and $N_6 = 1$. The generators of $\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ act on three complex coordinates of $\mathbb{C}^3$ as follows:

$\mathbb{Z}_{N_5}$	$\mathbb{Z}_{N_6}$
$z_1 \to z_1 e^{+2\pi i/N_5}$	$z_1 \to z_1$
$z_2 \to z_2 e^{-2\pi i/N_5}$	$z_2 \to z_2 e^{+2\pi i/N_6}$
$z_3 \to z_3$	$z_3 \to z_3 e^{-2\pi i/N_6}$

(3.1)

The orbifold transformation as an action on complex coordinates is embedded in a U(3) subgroup of the possible SO(6) holonomy. Since the phases add to zero, it is actually SU(3). The orbifold is a singular non-compact Calabi-Yau three-fold, preserving one quarter of the original supersymmetry. For $(N_5, N_6) = (N, 1)$, the orbifold action is in SU(2) and the resulting A_{N-1} singularity is a non-compact K3 manifold preserving one half of the original supersymmetry.

What is the world-volume theory of D-branes probing such an orbifold? The gauge group and matter representations describing k D-branes will be depicted in the quiver diagram: every node of the quiver diagram denotes a gauge group and the arrows connecting the nodes denote the $(\mathbf{K}, \bar{\mathbf{K}}')$ bifundamental representation of those groups $\mathrm{U}(K)$ and $\mathrm{U}(K')$. In our case all $K = K' = k$ and, because the overall U(1) decouples, we will discuss $\mathrm{SU}(k)$ groups only.

D-branes that can escape from the singularity at the origin must be accompanied by their $\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ mirror images on the covering space (where we count also the original D-branes); this number equals the order of the discrete group. If we want to describe k D-branes, it is therefore useful to start with a $\mathrm{U}(N_5 N_6 k)$ group and project onto states invariant under the orbifold action. The orbifold acts on the $\mathrm{U}(N_5 N_6 k)$ Chan-Paton labels in addition to the obvious action of (3.1) on the 6 scalars $z_i, \bar{z}_i$ and their fermionic partners. For this action on the group indices, we have to represent every element of $\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ by an element of the gauge group $\mathrm{U}(N_5 N_6 k)$. In our case of an abelian orbifold, the elements of the group are represented by block diagonal matrices in the $\mathrm{U}(k)^{N_5 N_6}$ subgroup. The vector multiplets have to be invariant under this action and therefore precisely the ones belonging to this $\mathrm{U}(k)^{N_5 N_6}$ group survive. Labeling those $N_5 N_6$ $\mathrm{U}(k)$s by $\mathrm{U}(k)_{l,m}$ $l = 1 \ldots N_5$ and $m = 1 \ldots N_6$ the nodes form a lattice as in figure 2.

For the scalars the geometric action has to be compensated by a non-trivial action on the Chan-Paton factors. The scalars therefore live in bifundamental representations. Consider e.g. the matrix Z_1 that carries the information about the positions of the D-branes in z_1. The physical action of the generator of $\mathbb{Z}_{N_5}$ adds a phase $\exp(+2\pi i/N_5)$. It must be cancelled by an action on the quiver nodes. The correct action is the shift of the nodes by one in the horizontal direction. Therefore the field Z_1 transforms in the direct sum of a fundamental representation and the anti-fundamental representation of the group which is to the right of the first one, one of the H fields in figure 2. Similarly Z_3 carries the links V in the vertical direction and Z_2 the diagonal fields D. The fermions follow by supersymmetry and complete H, V and D into chiral multiplets.

Our superconformal field theories are obtained as the $l_s \to 0$ limit of these orbifolds. We can match the parameters of the orbifold to the field theory by comparing physical

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states. The KK modes of the field theory correspond to fundamental strings stretching between the D-branes and their mirror images as in figure 3. The masses of these modes are the string tension times the length of the string, reproducing the field theory KK spectrum. The lightest KK mode, with mass $1/R$, corresponds to the fundamental string stretching between nearest neighbor D3-brane mirrors. The top of the KK tower, with mass $1/a$, corresponds to the string which stretches close to the fixed point. Similarly the magnetic and dyonic field theory modes correspond to the D- (more generally (p,q)) strings stretching between the D3-brane mirrors.

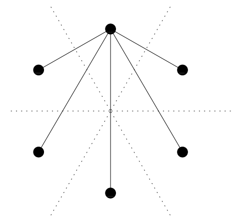

Figure 3: The W-bosons of the quiver gauge theory from strings stretched between D-branes and their mirror images under the orbifold action.

3.2 The $(2,0)$ theory

We wish to show that the four-dimensional $\mathcal{N}=2$ circular theory of the previous section on the Higgs branch, in the limit $N \to \infty$ holding the mass of the lightest KK mode m and its dual M fixed is $(2,0)_k$ theory compactified on a torus with radii $R_5 = 1/m$ and $R_6 = 1/M$. We have realized this four-dimensional field theory as the $l_s \to 0$ limit of k D3-branes on a $\mathbb{C}^3/\mathbb{Z}_N$ orbifold. This string theory has four parameters: l_s, g_s, N and the distance d of the D3-branes from the orbifold fixed point. The field theory parameters are related to these by

$$\begin{aligned} m &= \frac{d}{N l_s^2} \equiv \frac{1}{R_5} \\ M &= \frac{d}{N l_s^2 g_s} \equiv 1 R_6 \,. \end{aligned} \tag{3.2}$$

Therefore we consider this orbifold theory in the limit $l_s \to 0, N \to \infty$ holding g_s and $d/(N l_s^2) = 1/R_5$ fixed. The length of the stretched strings which correspond to the KK modes in the field theory must be "sub-stringy": $d \ll l_s$. The orbifold then reduces to the field theory at energies $E \ll 1/l_s$. Probing the physics at energy E involves fluctuations on the orbifold over distances of size $\Delta x \sim E\, l_s^2$. This implies $\Delta x/d \sim E R_5/N$. Thus in the large-N limit the D3-branes only fluctuate over distances small compared to d. Consequently we may approximate the $\mathbb{C}^3/\mathbb{Z}_N$ orbifold as $\mathbb{R}^5 \times S^1$ where the radius of the circle is $r = d/N \sim l_s^2/R_5$ plus corrections which vanish as $N \to \infty$. Figure 4 represents the "straightening" of the space near the D3-branes. More formally, choosing coordinates

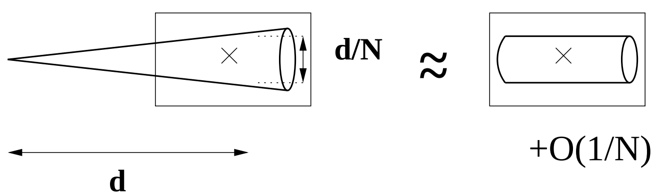

Figure 4: In the large-N limit the orbifold geometry locally degenerates into a cylinder.

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on $\mathbb{C}^3$ as $z_1 = r_1 \exp(i\theta + i\phi/N), z_2 = r_2 \exp(i\theta - i\phi/N)$ and z_3, the orbifold metric is flat with the periodic identifications $\theta \to \theta + 2\pi, \phi \to \phi + 2\pi$. Expanding $r_1 = d/\sqrt{2} + x_1, r_2 = d/\sqrt{2} + x_2, \theta = x_3/d$ the metric becomes

$$ds^2 = dz_3 d\bar{z}_3 + dx_1^2 + dx_2^2 + dx_3^2 + \left(\frac{d}{N}\right)^2 d\phi^2 \tag{3.3}$$

up to corrections of order $\Delta x/d \sim ER_5/N$ which vanish in the large-N limit. In this picture the enhancement to 16 supercharges is manifest—supersymmetry breaking is associated with the orbifold fixed point which can not probed in our limit.

In the small l_s limit the radius r is sub-stringy—a better description is obtained by taking the T-dual to turn this into a large circle with radius R_5.footnoteAt finite N, where the orbifold fixed point can still be probed, this T-duality involves NS5-branes corresponding to the orbifold fixed point [36] The D3-branes become D4-branes wrapped around this large circle, while the string coupling becomes $g_s' = g_s R_5/l_s$. As l_s shrinks to zero, the string coupling becomes large—a better description is obtained by using M-theory. The wrapped D4-branes become M5-branes wrapped on a torus with radii R_5 and $R_6 = g_s' l_s = g_s R_5$. The 11-dimensional Planck length is $l_p^3 = l_s^3 g_s' = l_s^2 R_6$. As we take the limit $l_s \to 0$ the 11-dimensional Planck length goes to zero. This is the definition of the $(2,0)_k$ SCFT in 6 dimensions compactified on a torus.

3.3 Little string theory

The $\mathcal{N} = 1$ toroidal quiver is similar to the $\mathcal{N} = 2$ circular case. We have realized this four-dimensional field theory as the $l_s \to 0$ limit of k D3-branes on a $\mathbb{C}^3/\mathbb{Z}_{N_5} \times \mathbb{Z}_{N_6}$ orbifold. The field theory obtained from the orbifold at finite l_s is not precisely the superconformal theory described previously. The orbifold matches to our toroidal quiver field theory with the boundary condition $\lambda = G$ at the scale $1/l_s$. As we take the limit $l_s \to 0$ the field theory flows to the superconformal fixed line. The continuum limit in this case is $N_{5,6} \to \infty$ holding the mass scales of the lightest KK modes $m_{5,6}$ and the dual winding modes $M_{5,6}$ fixed. This string theory has six parameters: $l_s, g_s, N_{5,6}$ and the distances $d_{5,6} = |z_{1,3}|$ of the D3-branes from the orbifold fixed point. The field theory parameters are related to these by

$$\begin{aligned} m_{5,6} &= \frac{d_{5,6}}{N_{5,6} l_s^2} \equiv 1/R_{5,6} \\ M_{5,6} &= \frac{d_{5,6}}{N_{5,6} l_s^2 g_s} \,. \end{aligned} \tag{3.4}$$

Thus we consider the orbifold theory in the limit $l_s \to 0, N_{5,6} \to \infty$ (with fixed N_5/N_6) holding g_s and $d_{5,6}/N_{5,6}$ fixed. The length of the stretched strings which correspond to the KK modes in the field theory must be "sub-stringy": $d_{5,6} \ll l_s$. The orbifold then reduces to the field theory at energies $E \ll 1/l_s$. Again, probing the physics at energy E involves fluctuations on the orbifold over distances of size $\Delta x \sim E\, l_s^2$, implying $\Delta x/d_{5,6} \sim ER_{5,6}/N_{5,6}$. Thus in the large-$N$ limit the D3-branes only fluctuate over distances small compared to $d_{5,6}$. Consequently we may approximate the $\mathbb{C}^3/\mathbb{Z}_N$ orbifold as $\mathbb{R}^4 \times T^2$ where the radii of the T^2 are $r_{5,6} = d_{5,6}/N_{5,6} \sim l_s^2/R_{5,6}$ plus corrections which vanish as $N_{5,6} \to \infty$. In the small l_s limit the radii $r_{5,6}$ are sub-stringy—a better description is obtained by T-dualizing to turn these into two large circles with radii $R_{5,6}$. The D3-branes become D5-branes wrapped around this large torus, while the string coupling becomes $\tilde{g}_s = g_s R_5 R_6 / l_s^2$. As l_s shrinks to zero, this string coupling $\tilde{g}_s$ becomes large—a better description is obtained by S-dualizing, giving an NS5-brane wrapped on the same torus, but with a new string coupling $g_s' = 1/\tilde{g}_s = l_s^2/(g_s R_5 R_6)$ and a new string length $l_s' = l_s\sqrt{\tilde{g}_s} = \sqrt{g_s R_5 R_6}$. As we take the limit $l_s \to 0$ the new string coupling $g_s' \to 0$ while the new string length l_s' remains fixed. This is the definition of the $(1,1)$ little string theory.

4. Deconstructing M-theory?

It is natural to attempt to extend our technique to deconstruction of other non-gravitational theories. A more interesting challenge is the deconstruction of M-theory itself. A partial realization of such a deconstruction proceeds as follows. In type-IIB string theory, consider k NS5-branes on a $\mathbb{C}^2/\mathbb{Z}_N$ orbifold. In the decoupling limit $g_s \to 0$ with l_s fixed, this is a $(1,0)$ little string theory. At distances larger than l_s, this theory reduces to a $(1,0)$ six-dimensional supersymmetric quiver gauge theory with the same quiver diagram as figure 1, with N additional tensor multiplets. Along the Higgs branch of this theory, we can hope to generate a *seventh* dimension of fixed radius R_7. Since there are no known non-gravitational UV completions of 7-dimensional supersymmetric gauge theories, we expect this to result in a gravitational theory. The analysis mirrors our $(2,0)$ string theory construction. We first S-dualize to obtain k D5-branes on the same orbifold, with a new string coupling $\tilde{g}_s = 1/g_s \to \infty$, and a new string length $\tilde{l}_s = l_s\sqrt{g_s} \to 0$. As before, the orbifold degenerates into a cylinder with radius $r \sim \tilde{l}_s^2/R_7$ smaller than the string length. A better description is obtained by T-dualizing to turn this into a large circle with radius R_7. The D5-branes become D6-branes wrapped around this large circle, while the string coupling becomes $g_s' = \tilde{g}_s R_7/\tilde{l}_s$ and the string length is unchanged $l_s' = \tilde{l}_s$. Since $g_s' \to \infty$, a better description is M-theory with radius $R_{11} = g_s' l_s' = R_7/g_s \to \infty$, and 11-dimensional Planck length $l_p^3 = g_s'(l_s')^3 = l_s^2 R_7$ staying fixed. The k wrapped D6-branes become an A_{k-1} singularity. The seven-dimensional gauge coupling of the theory living on the A_{k-1} singularity is $g_7^2 = R_7 l_s^2 = l_p^3$. Therefore, the bulk 11-dimensional gravity does not decouple from the seven-dimensional gauge dynamics. Finally we can decompactify the seventh dimension $R_7 \to \infty$ holding $g_7^2 = l_p^3 = R_7 l_s^2$ fixed. Thus, we claim that the deconstruction of a seventh dimension beginning with a $(1,0)_k$ little string theory describes M-theory on an A_{k-1} singularity with fixed 11-dimensional Planck length. We have therefore deconstructed a fully gravitational theory using non-gravitational little strings. This is similar in spirit to the approach of [22]. It would be interesting to further deconstruct the little strings to obtain a completely field-theoretic description.

5. Conclusions and discussion

The exploration of four-dimensional field theories characterized by theory spaces has revealed surprising new phenomena. In some cases, non-gravitational extra dimensions arise from theory space at low energies [1], but more generally there are new features intrinsic to theory space itself. This realization opens up two complementary avenues of exploration. In one direction, the rich dynamics of theory space can provide new approaches to beyond the standard model physics, such as stabilizing the electroweak scale [23], lowering the scale of gauge coupling unification [24], breaking supersymmetry [25], and other applications [26]–[30]. In another, extra-dimensional physics can be explored on a firm footing in a broader context. In this paper, we have connected deconstructed dimensions with string theory. We have argued that, with sufficient supersymmetry, certain limits of local, *four*-dimensional supersymmetric field theories described by theory spaces become $(2,0)$ SCFT and little string theory in *six* dimensions.

It would be interesting to understand many aspects of our correspondence in more detail. For instance, the little string theory is known to have a Hagedorn transition [31, 32]—how is this reflected in the field theory? In our four-dimensional theory at the origin of moduli space, the conformal invariance is enough to ensure that the free-energy scales as $N_5 N_6 T^4$. Even out on the Higgs branch where the extra dimensions are formed, this should be the behavior of the free-energy at temperatures above the Higgsing scale, that sets the little string tension. Since the free-energy diverges as $N_{5,6} \to \infty$, we might expect a limiting temperature set by the string tension. However a direct verification of an exponential growth of the number of states with mass set by the little string scale would be far more convincing. Although the little strings wrapped around the extra dimensions are likely four-dimensional instantons "lifted" into the theory space, it would be nice to construct the little strings explicitly as field theory solitons. Doing so is important for elucidating issues such as the tension between locality and T-duality [8]. It would also be interesting to understand the relation of our constructions with matrix theory [7].

Our construction is based on orbifold realizations of our four-dimensional field theories. In the large-N limit these orbifold geometries are only probed in a way that is insensitive to the orbifold fixed points, so that the relevant geometry is well-approximated by compact circles cross flat space. String dualities can then be used to relate these field theories to known sectors of string theory. There are likely deconstructions of other non-gravitational theories in five and six dimensions, such as theories with less supersymmetry, ॐ [33] and NCOS [34, 35] theory, *etc.* Especially interesting would be the complete deconstruction of a gravitational theory. We have indicated a first step in the deconstruction of M-theory, starting with a non-gravitational little string theory in six dimensions.

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